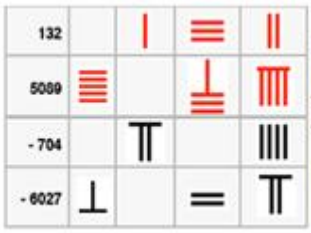
CLASS NOTES 5: NEGATIVE NUMBERS

Negative numbers are pretty cool. Even though mathematics is about 5000 years old, negative numbers only came on the scene in the second half of that time.



In 200 BCE the Chinese number rod system represented positive numbers in Red and Negative numbers in black. An article describing this system can be found here . These were used for commercial and tax calculations where the black cancelled out the red. The amount sold was positive (because of receiving money) and the amount spent in purchasing something was negative (because of paying out); so a money balance was positive, and a deficit negative.

(It's interesting that today, we use the exact ***opposite*** colors for positive and negative when doing finances. In fact, legend has it that's how "Black Friday" got its name…because that's when stores sold a lot of merchandise and got back "in the black."[1])

Let's start with some arithmetic problems – find answers to the following.

1. **10 – 1**
2. **10 – 2**
3. **10 – 3**

Just a few more…

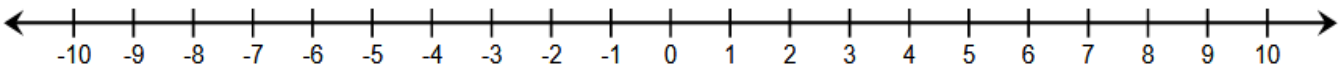
4. **10 – 8**
5. **10 – 9**
6. **10 – 10**

(*before we go any further – let's watch this little video*!)

And, now…

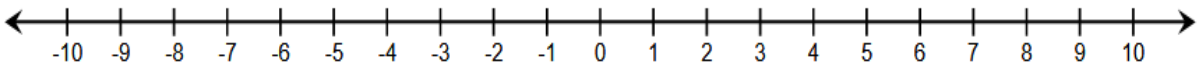
7. **10 – 11**

8. In that last one (10 – 11), what kind of number did you get? ***Why*** did you get that? Write a bit about what that number means.



(*let's do a video check-in here*!)

[1] Of course, "Black Thursday" was the date in history that set off the first Great Depression (and also the one in March 2020), so…



Now, to the main topic of this unit: let's see how to do some arithmetic with positive AND negative numbers. And we'll use that number line again.

9. **5 + 3**

<u>Negative Numbers Rule 1</u>: When you add a positive number to any number, move to the right on the number line as many spaces as the positive number tells you to.

10. **-5 + 3**

11. **-3 + 3**

12. **5 + (-3)** **(ask yourself: how about when you add a negative number?)**

<u>Negative Numbers Rule 2</u>: When you add a <u>negative</u> number to any number, move to the <u>left</u> on the number line as many spaces as the <u>negative</u> number tells you to.

13. **-5 + (-3)**

14. **-5 – 3**

15. Why are those last two the same?

<u>Negative Numbers Rule 2(a)</u>: Adding a negative number is the same as subtracting a positive number.

(*let's check in with a video*!)

16. **10 – (-5)** **(ask yourself: how about when you subtract a negative number?)**

Negative Numbers Rule 2(b): Subtracting a negative number is the same as adding a positive number.

17. **-10 – (-5)**

(*let's check in with a video*!)

Now let's try some with multiplication!

18. **-2 * 3**

19. **4 * (-8)**

20. (*choose one*) When you multiply a positive number by a negative number, the result is

positive negative

21. Why is this the case? Does it matter whether the positive or the negative number comes first?

(*let's check in with a video*!)

For the next few questions, if you're not sure which choice to make, experiment with various numbers on your calculator.

22. (*choose one*) When you multiply a negative number by a negative number, the result is

positive **negative**

23. Why?

24. (*choose one*) We've seen that multiplying two numbers of the same sign (that is, both positive or both negative) results in a positive number. Also, we've seen that multiplying two numbers of different signs results in a negative number. Does division follow these same sign rules? Experiment with different numbers and see!

Yes **No**

25. Why (or why not)?

26. (*choose all that apply*) Back in #21, you noticed that when you multiplied two numbers with different signs, the result was negative whether the positive or the negative number came first. In general, which operations does order **matter** for? In other words, for which operations do you get a **different** result if you switch around the numbers?

Addition **Subtraction** **Multiplication** **Division**

(*let's check in with a video*!)

Finally, let's look at some combinations of operations involving negative numbers. As always, feel free to use technology!

27. Here's a type of expression you'll encounter in MTH 105. Go ahead and evaluate it (again, using tech if you want)!

$$5*(-1) + 6*(1)$$

28. Here's another MTH 105-type problem.

$$0.5071*(-1) + 0.4929*(1)$$

29. Try it again – with ***units***.

$$0.5071*(-\$1) + 0.4929*(\$1)$$

You may have noticed that #27 and 28 involve numbers with the same combination of signs (positive times negative, plus positive times positive) …but the result in #28 was positive and the result in #29 was negative. Let's explore that a little more.

30. Evaluate this one:

$$0.003*(\$109) + 0.043*(\$9) + 0.213*(\$1) + 0.741*(-\$1)$$

31. Now compare it to ***this*** one:

$$0.003*(\$54) + 0.043*(\$4) + 0.213*(\$0) + 0.741*(-\$1)$$

Now, those might just seem like weird math exercises. However, they're actually both expressions that calculate the average payouts for a scratch-it game that's played in New York State. See how one was positive and one was negative? A negative payout means people that buy the lottery tickets, on average, lose money. That makes sense! That's why lotteries work. A ***positive*** payout means the people who buy the tickets are making money, on average – bad business for the lotteries (in fact, two kids figured out the math error that the lottery commission made with this game and each made about $250,000 in 4 weeks).

32. In all of the last five problems, once we were done with the multiplication, we ended up adding some positive and negative numbers together. Sometimes our result was positive and sometimes it was negative. Now, let's keep it simple and think about adding only two signed numbers together. How do you know whether your answer will be positive or negative?

(*let's check in with a video*!)