Lab 8: Expected Value

I'm going to describe two dice games to you here. Check 'em out!

- **Game #1: Roll a single die one time. You win the number of pieces of candy that shows on the dots of the die.**

- **Game #2**: Roll a single die one time. If you roll a 2, 3, 4, 5, or 6, you win ***twice*** that number of pieces of candy. However, if you roll a 1, you need to give back 15 pieces of candy (or go into debt, if you don't have that many). In other words, you "win" *negative* 15 pieces of candy.

1. (**1 point**) If you had to play one of these games exactly **one** time, which would you choose? Why?

2. (**1 point**) If you had to play one of these games **many, many** times, which would you choose? Why? And it's cool to pick the same game for both of these!

Now, I'll have you play each game 10 times! Please keep track below in the "…this many times" column. If you are playing at home and don't have a die, this website allows you to roll a virtual die.

In case you need an explanation of what I mean by "play each game 10 times", watch this video!

Game #1

If you roll a…	…then you win:	This is how many times you rolled that number!
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	

Game #2

If you roll a…	…then you win:	This is how many times you rolled that number!
1	-15	
2	4	
3	6	
4	8	
5	10	
6	12	

3. (**1 point**) (**w**) On average, how many pieces of candy did you win ***per game*** in game #1?

4. (**1 point**) (**w**) On average, how many pieces of candy did you win per game in game #2?

(*check out this video if you get stuck on 3 and 4*)

5. (**2 points**) Which game was better for you, on average (meaning, "on average, you won more candy")? Does that game correspond to your answer in problem #2?

To be fair, 10 trials isn't a very good sampling of how well a game based on random results occurs. Ideally, we should be able to play this game many, many times and see what the underlying behavior is as the game's results reveal themselves to us—much like we did with R2D2 at the beginning of the term.

The downside, of course, is that playing this game over and over is tedious. But not to worry! We've created a simulator that "plays the game" many, many times: check out this video to see it in action! Let's use the results from that video to answer some questions:

6. (**3 points**) (**w**) On average, how many pieces of candy did the simulator win per game in Game #1? Show me everything you do!

7. (**3 points**) (**w**) On average, how many pieces of candy did the simulator win per game in Game #2? Same note on work!

Now, you might not have noticed this near the end of the video, but those 2nd columns in each table were hiding something cool! I let the simulator run a whole lot more, and this is what I saw:

Cumulative Results

Game #1

What you won	How Many Times
1	3196
2	3161
3	3044
4	3058
5	3029
6	3057

Game #2

What you won	How Many Times
-15	3190
4	3155
6	3032
8	3051
10	3024
12	3048

See anything interesting about those "How Many Times" columns?

If not, take a look at this slightly different look at that data!

Cumulative Results

Game #1		Game #2	
What you won	**% of the time**	**What you won**	**% of the time**
1	17.2%	-15	17.2%
2	17.0%	4	17.1%
3	16.4%	6	16.4%
4	16.5%	8	16.5%
5	16.3%	10	16.3%
6	16.5%	12	16.5%

8. (**2 points**) Tell me how we got those percentages! You'll most likely have to look back to the previous screen shot.

9. (**2 points**) Now, explain to me how you could have known those %'s were all going to be really, really close to 16.666%...without ever using the simulator. 😊

And that means that we could have used a very, very handy shortcut to figure out the averages of each game…***if*** we had recognized those rates! Check this out!

The average, or <u>expected value</u>, of an experiment can be found by

a) Multiplying each payout by its rate of occurrence, and then

b) Adding all of those products together.

The resulting number is what you will "earn", on average, when you do the experiment.

I'll show you how to use this expected value rule for game #1 in this video!

10. (**3 points**) (**w**) OK! You do the expected value of game 2! **<u>Hint</u>**: look back at #7 – these should be roughly the same answer!

So, now, let's regroup and remember what these averages ***mean***: they mean that, on average, you will win 3.5 pieces of candy per game from game #1, and 4.2 pieces of candy per game in game #2. So, over the long run, Game #2 is better for the player!

Now, here's a new game:

Game #3

If you roll a…	…then you win…
1	1
2	-2
3	3
4	-4
5	5
6	-6

11. (**1 point**) What's the expected value for ***this*** game?

12. (**2 points**) Remind me, again, what your Game #3 average ***means***?

13. (**1 points**) So, on average, which of the 3 games is the best for the player?

14. (**2 points**) (**w**) Make up your own dice game! Fill in the second column below to describe the payouts (make sure to use a combination of positive and negative numbers to keep the game exciting), and then calculate its expected value in the space below.

If you roll a…	…then you win…
1	
2	
3	
4	
5	
6	