Lab 1: Multi-Sided Dice

Today, we're going to dip our toes into some concepts you'll be exploring a lot more in our next lab. Rolling a die (the singular form of "dice") is a very simple **experiment.** And it has a certain number of **outcomes**, all of which are equally likely.

Suppose you have a typical collection of multi-sided dice—e.g., one you'd buy for tabletop gaming. This collection (shown to the right) contains your good old-fashioned 6-sided die, but also 4-sided, 8-sided, 10-sided, 12-sided, and 20-sided dice.

Dice are really just analog RNG's: **R**andom **N**umber **G**enerators. If you need to pick a random choice from a certain number of options, a die (or more than one die) is a great way to do it!

And dice are great if you need whatever number of outcomes they actually have on them (4, 6, 8, 10, 12, or 20). But what if, say, you needed a 3-sided die (and you don't want to buy this fancy one)?

One solution is to just use the 6-sided one, but "relabel" the sides as follows:

If you roll a…	…which happens this often…	…then call it…	…and that letter happens…
1	$\frac{1}{6}$ **of the time**	**A**	$\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$ **of the time**
2	$\frac{1}{6}$ **of the time**		
3	$\frac{1}{6}$ **of the time**	**B**	$\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$ **of the time**
4	$\frac{1}{6}$ **of the time**		
5	$\frac{1}{6}$ **of the time**	**C**	$\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$ **of the time**
6	$\frac{1}{6}$ **of the time**		

And there you have your "3-sided die"!

1. **(2 points)** Using one of the other dice in that image at top right (besides the 6-sided one!), explain how to "relabel" it so it could also be a "3-sided die". No need to fill out an entire chart like I just did – that was just to be clear about the math. Just explain in words.

2. **(2 points)** Using one of the dice in Figure 1, explain how to relabel it so it could also be a "5-sided die".

3. **(2 points)** Using one of the **other** dice in Figure 1 (besides the one you used in #2), explain how to relabel it so it could *also* be a "5-sided die".

So we've been able to model 3-sided and 5-sided dice using our pack in Figure 1. But what about ***other*** numbers of sides?

4. **(3 points)** Why can't we model a 7-sided die with this pack in the same way we did with the 3- and 5-sided ones?

Now, let's try something different: we're going to use those dice in Figure 1 to simulate a 2-sided **coin**. But not just any old coin—I never said I wanted both sides to be equally ***likely***! In fact, I want a coin where "heads" comes up **three** times as often as "tails".

5. **(3 points)** One way to do this is to use the 4-sided die. Call 1, 2, and 3 "heads" and 4 "tails". Explain why, indeed, that assignment on a 4-sided die would satisfy the requirement in the paragraph above ("heads" comes up **three** times as often as "tails").

6. **(2 points)** Which other dice in our pack could be used to achieve the same "three times as likely to get heads as tails" result? List all that apply.

4-sided	6-sided	8-sided	10-sided	12-sided	20-sided

7. **(2 points)** Look back to the ones you didn't circle. Why ***can't*** you do it with those? (Another way of looking at it is to ask, "Why ***can*** you do it with the ones you circled?")

8. **(2 points)** OK, now I want a coin where "heads" comes up **five** times often as "tails". Which dice could you do it with? List all that apply.

4-sided	6-sided	8-sided	10-sided	12-sided	20-sided

9. **(2 points)** One more! I want a coin where "heads" comes up **nine** times often as "tails". Which dice could you do it with?

4-sided	6-sided	8-sided	10-sided	12-sided	20-sided

10. **(1 point) For** that last requirement ("heads nine times as likely as tails"), give one more die, *besides* those listed, that you ***could*** use (if you could manufacture it) to achieve the same ratio of heads and tails!

11. **(4 points)** Suppose that I say to you, "I want to pick a die to simulate a coin. For this coin, I need to have 'heads' come up h times as often as 'tails'." Explain how you can decide which dice would work, based on the value of h. I'm not necessarily looking for a specific formulaic answer; feel free to answer in a sentence or two. Here's a short YouTube playlist (2 videos) to help if you get stuck!